Radiative energy transfer

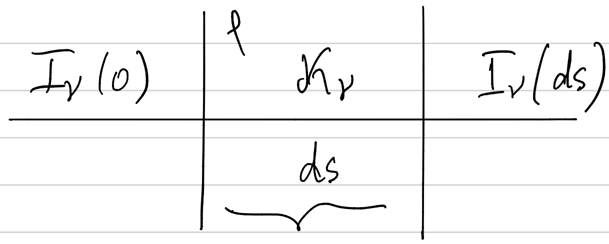
Radiative Energy Transfer

Extinction coefficient

When a radiation pass through a medium, it can be attenuated in proportion of I_ν and column density $\rho\, ds$

Consider $I_\nu(0)$ going through ds of density ρ

$\kappa_\nu \equiv$ opacity



$[\kappa_\nu] \equiv cm^2 g^{-1}$

The amount of absorbed radiation : $dI_\nu = -\kappa_\nu \rho I_\nu\, ds$

We shall see that κ_ν results from different physical processes (wait for next chapter ...)

$$\frac{dI_\nu}{ds} = -\kappa_\nu \rho I_\nu$$

if κ_ν et ρ are constants : $I_\nu(s) = I_\nu(s_0)\, e^{-\kappa_\nu \rho (s - s_0)}$

Mean free path :

$$\ell = \frac{1}{\kappa \rho} \qquad I_\nu(s) = I_\nu(s_0)\, e^{-\frac{(s-s_0)}{\ell}}$$

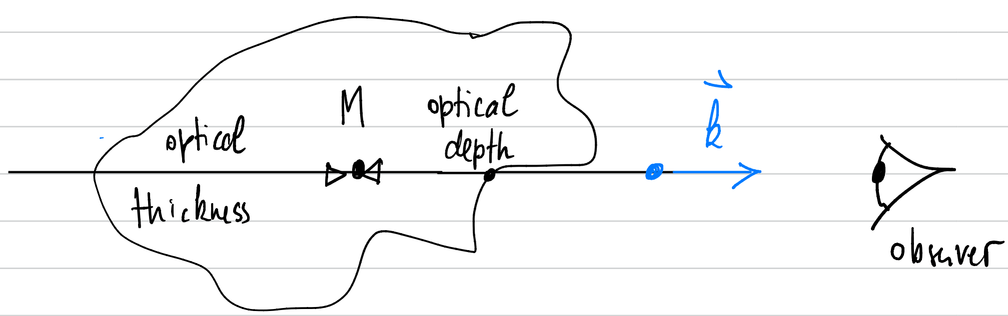
stellar interiors : $0.01 \lesssim \ell \lesssim 1$ cm.

After travelling a length ℓ, the intensity is decreased by a factor e.

1 μm	at the center of $\odot$	
1 cm	at	T = 25000 K
1 km	at	T = 10 000 K
100 km	at	T = 6000 K
10^3 km	at	T = 5000 K red giants

Optical depth and stellar thickness

optical depth $\quad \tau_\nu = \int_{s_0}^{s} \kappa_\nu \rho\, ds' \longrightarrow I_\nu(s) = I_\nu(s_0)\, e^{-\tau_\nu}$



extinction include a part of absorption and a part of scattering

$$K_\nu^{tot} = K_\nu^{scat} + K_\nu^{abs}$$

For a radiation with incident angle θ

Absorbed energy $\quad dU_\nu^{abs} = |dI_\nu|\, d\nu\, d\sigma \cos\theta\, d\Omega\, dt$

$$= K_\nu\, \rho I_\nu\, ds\, d\nu\, d\sigma \cos\theta\, d\Omega\, dt$$

$\rho\, ds\, d\sigma$ = mass element = dm [light crossing dm]

$$dU_\nu^{abs} = K_\nu I_\nu\, d\nu\, dm\, d\Omega\, dt$$

Scattered light scattered in direction θ'

$$dU_\nu^{scat} = K_\nu^{scat} I_\nu\, dm\, d\nu\, d\Omega\, dt \left[\frac{d\Omega'}{4\pi} p^{diff}(\cos\theta')\right]$$

p^{diff} = phase function such that $\displaystyle\int_{\Omega'} \frac{p^{diff}}{4\pi} d\Omega' = 1$

Mie phase function $\quad p = \frac{3}{4}(1 + \cos^2\theta)$

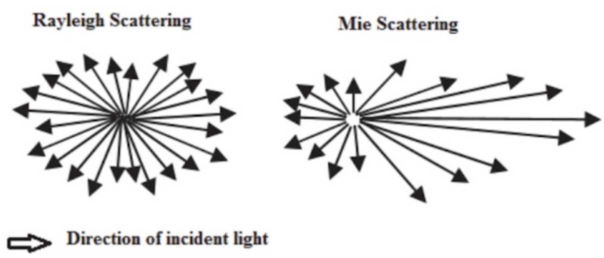
isotropic scattering $\quad p = 1$

Les mécanismes d'*absorption* et de *diffusion* sont décrits par les *théories de Mie et de Rayleigh*.

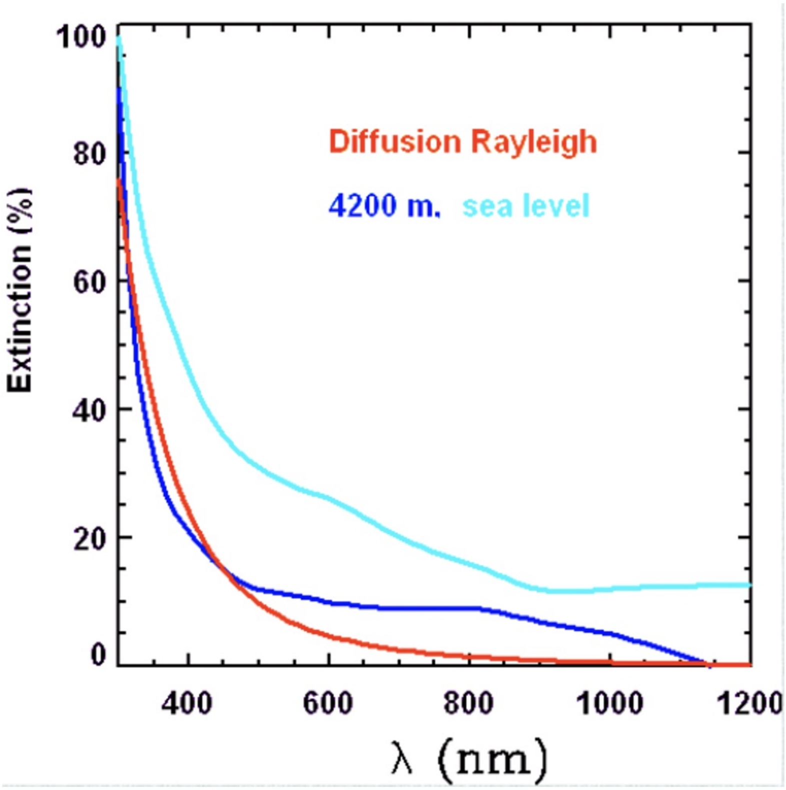
Ces deux théories sont une application des équations de Maxwell à des particules sphériques ou ellipsoïdales de petites tailles.

La *théorie de Rayleigh* s'applique à des particules d'un diamètre très inférieur à la longueur d'onde.

Lorsque le diamètre des particules est $\geq 0.1\boldsymbol{\lambda}$, le phénomène de diffusion est beaucoup plus complexe et il faut utiliser la *théorie de Mie*.

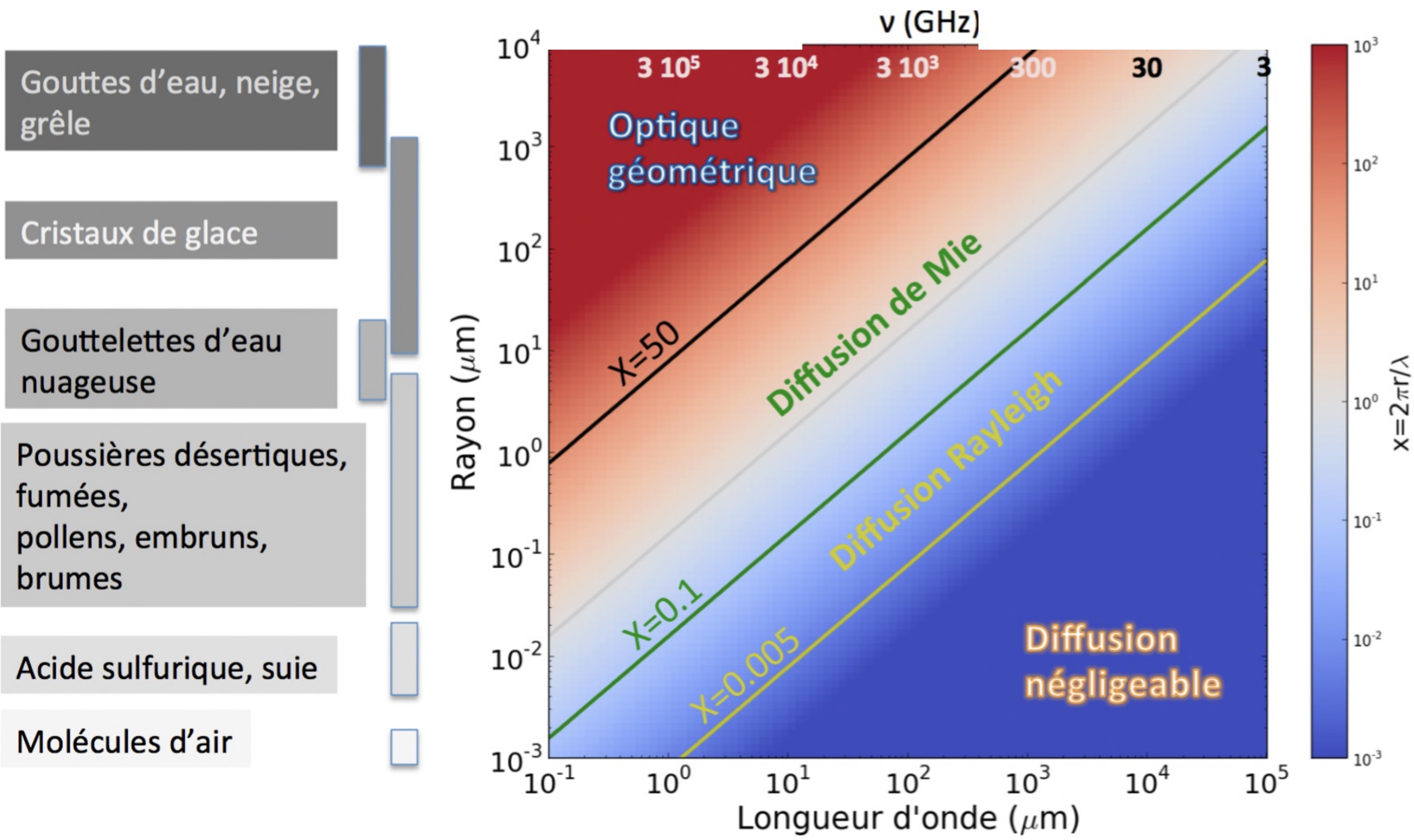


La diffusion Rayleigh varie comme $\boldsymbol{\lambda}^{-4}$
Elle est bien plus forte dans le bleu, à 400 nm, que dans le rouge à 650 nm. Ceci explique pourquoi le ciel est bleu la journée



Paramètre de Mie et diffusion

Paramètre de Mie:
$x = 2\pi r/\lambda$



visual domain

Albédo : $a_\nu = \dfrac{K_\nu^{diff}}{K_\nu^{diff} + K_\nu^{abs}}$

$$\Rightarrow dU_\nu^{diff} = \frac{U_\nu^{diff}}{U_\nu^{diff} + U_\nu^{abs}}$$

Absorption $\equiv$ conversion of radiative energie in thermal energy

Scattering $\equiv$ no conversion of energy, photons changes direction

Emission coefficient

A mass element can emit light (photons) per production of nuclear energy or as a black body (temperature)

soit par production d'énergie
de type corps noir (σ ~~tem~~ température = emissivity)

$$dU_\nu^{em} = j_\nu \, d\nu \, dm \, d\Omega \, dt$$

$$[j_\nu] \equiv W \, kg^{-1} \, steradian^{-1} \, Hz^{-1}$$

on can write $dI_\nu^{em} = j_\nu \rho \, ds$

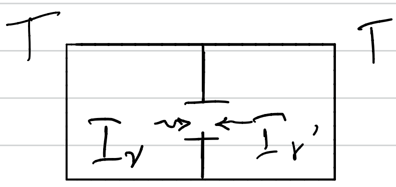
in analogy with $dI_\nu^{abs} = -K_\nu (I_\nu) \rho \, ds$

The Kirchhoff law

We consider a closed cavity, with walls at temperature T

The system is at thermal equilibrium

A each frequency there is equilibrium between emitted and absorbed energy



si le système est à ce qu'il finira par isolé, le rayonnement émis = le rayonnement reçu.

$\Downarrow$

irreversible transformation

$\rightarrow$ La 1ère loi de Kirchhoff ne peut être qu'une fonction dépendante de la température.

Equilibrium between emission and absorption

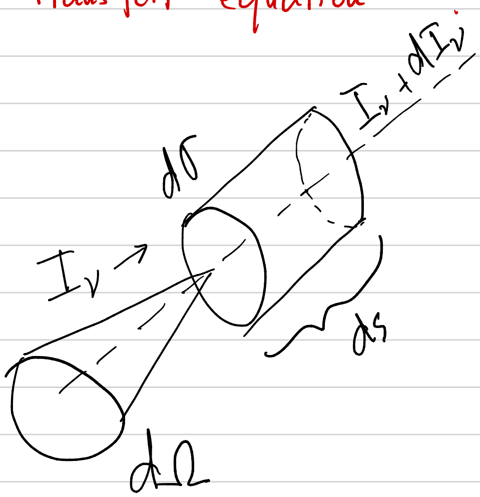
$$dU_\nu^{abs} = K_\nu I_\nu \, d\nu \, dm \, d\Omega \, dt$$

$$dU_\nu^{em} = j_\nu \, d\nu \, dm \, d\Omega \, dt$$

$$\Rightarrow K_\nu I_\nu = j_\nu \qquad I_\nu = \frac{j_\nu}{K_\nu}$$

independant of medium

Transfert equation



In all celestial bodies, there is transfert of from the center to the surface irrespective of the source of internal energy

ex: gravitational collapse (proto $*$)

fusion ($*$), fission (earth)

Variation of energy in dt : $dU_\nu = dI_\nu \, d\sigma \, d\nu \, d\Omega \, dt$

Emitted energy $dU_\nu^{em} = j_\nu \, \rho \, d\sigma \, ds \, d\nu \, d\Omega \, dt$

Absorbed energy $dU_\nu^{abs} = K_\nu I_\nu \, \rho \, d\sigma \, ds \, d\nu \, d\Omega \, dt$

Balance : $dU_\nu = dU_\nu^{em} - dU_\nu^{abs}$

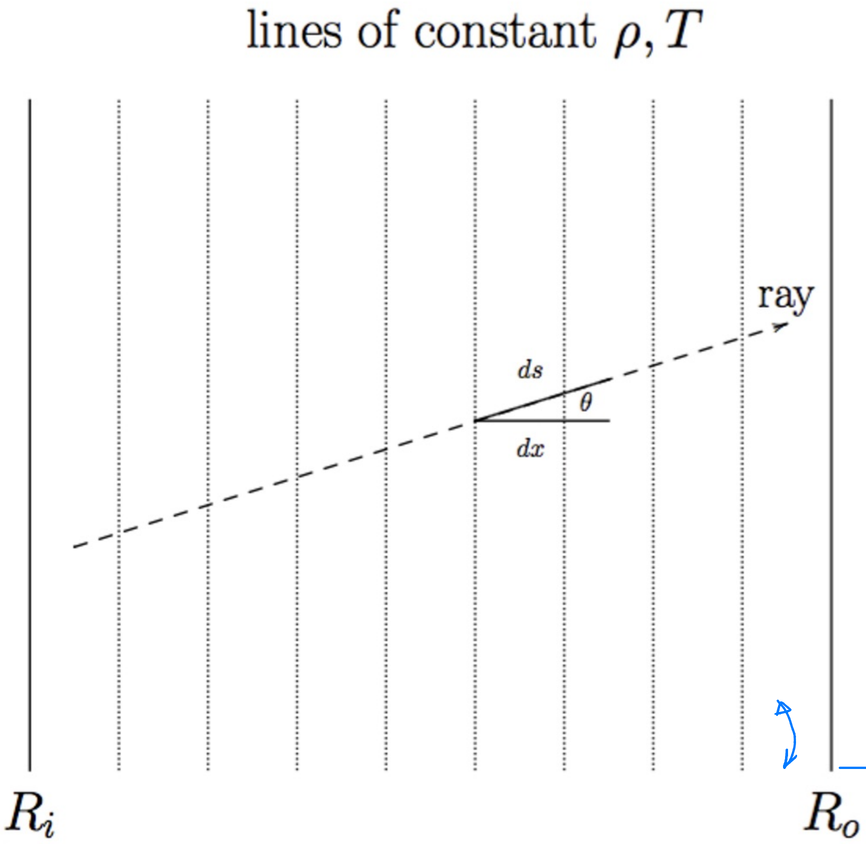
$$\frac{dI_\nu}{ds} = \rho \left(j_\nu - K_\nu I_\nu \right)$$

Equation of radiative transfert

Spherical geometry :

$$\frac{dP_{rad}}{dr} + \frac{1}{r}\left(3P_{rad} - u\right) + \frac{\rho K F}{c} = 0$$

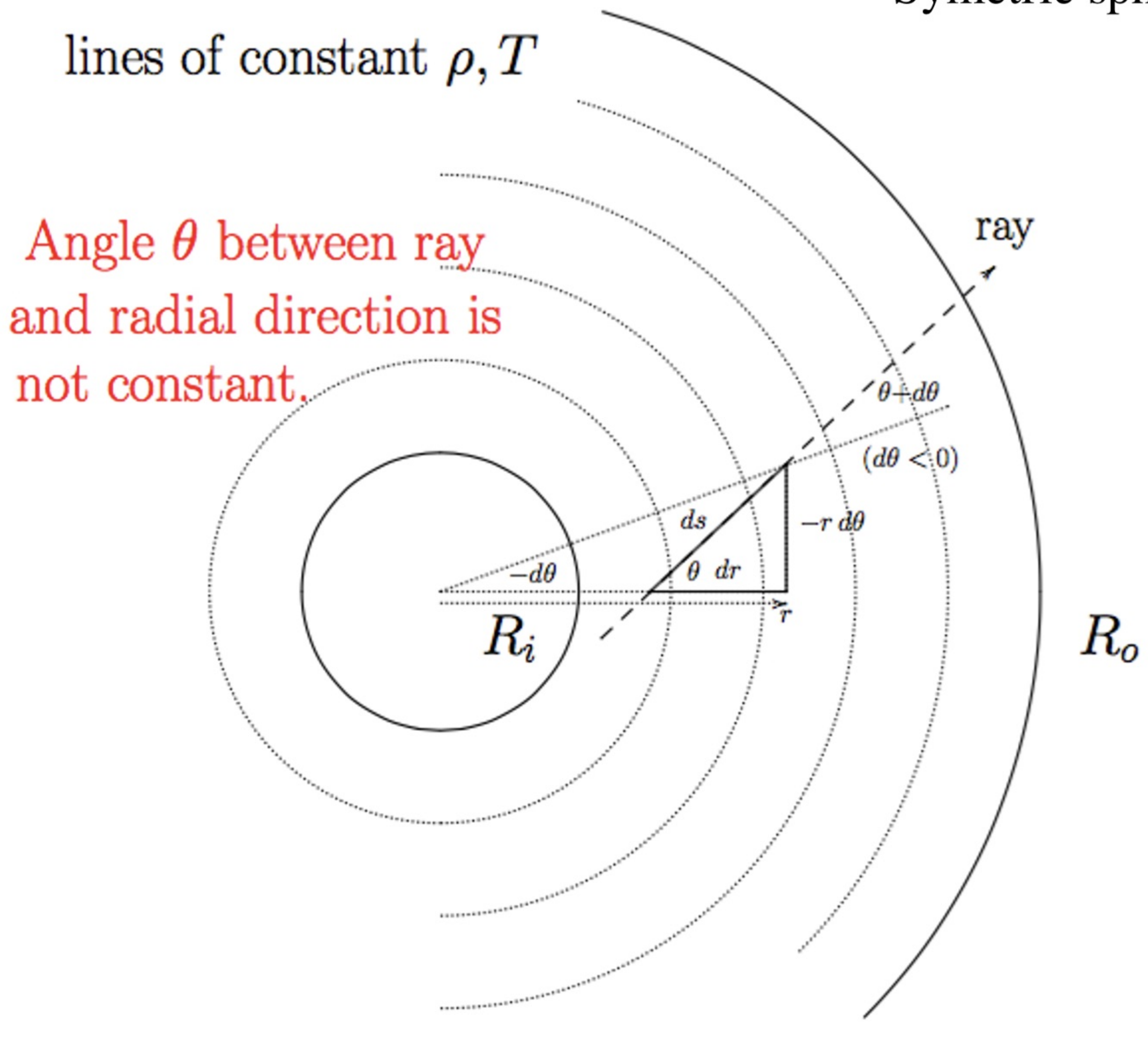
Plan parallèle



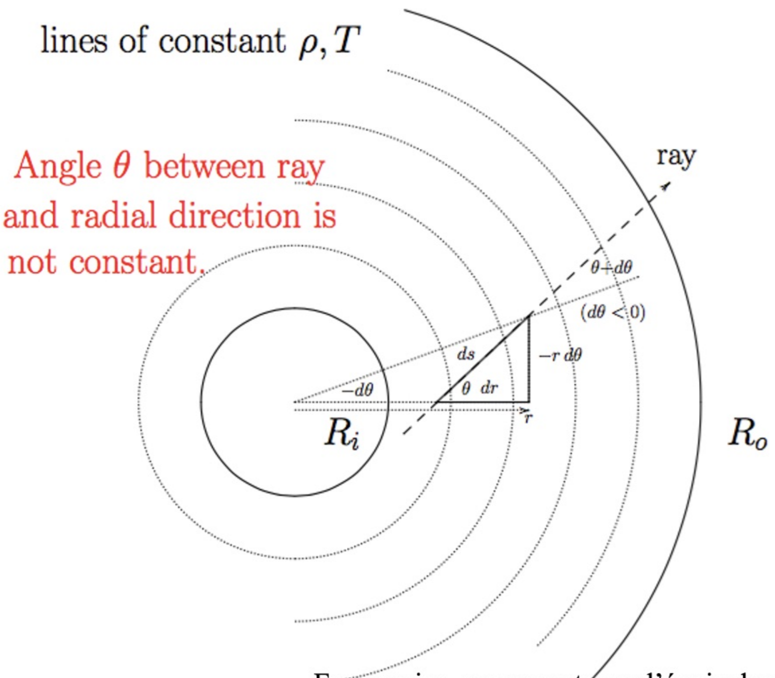
$$dx = \cos\theta\, ds = \mu\, ds$$

$$\frac{d}{ds} = \mu \frac{d}{dx}$$

Symétrie sphérique



Symétrie sphérique



$$\frac{d}{ds} = \frac{dr}{ds}\frac{\partial}{\partial r} + \frac{d\theta}{ds}\frac{\partial}{\partial \theta}$$

$$dr = \cos\theta\, ds = \mu\, ds \qquad \frac{dr}{ds} = \cos\theta$$

$$-r\, d\theta = \sin\theta\, ds \qquad \frac{d\theta}{ds} = \frac{-\sin\theta}{r}$$

En exercice, vous montrerez l'équivalence avec

$$\frac{\mathrm{d}P_{\mathrm{rad}}}{\mathrm{d}r} + \frac{1}{r}(3P_{\mathrm{rad}} - u) + \frac{\rho\kappa F}{c} = 0$$

Transfer equation in stellar interiors

LTE $\equiv$ Local Thermodynamic Equilibrium

$\equiv$ At each depth, the radiation field only depends on local T $\qquad I_\nu = B_\nu(T)$

LTE is satisfied if ℓ, photon mean free path $\ll$ scale of temperature variation

$$\frac{dT}{dr} \ll \frac{T}{\ell} \equiv \ell \ll \frac{T}{\frac{dT}{dr}}$$

- $\boxed{\text{How much is } \frac{dT}{dr}}$? $\qquad \left|\frac{dT}{dr}\right| \sim \frac{T_c}{R} \sim \frac{10^7 K}{7.10^{10} cm} \sim 10^{-4} K.cm^{-1}$

if $T \sim 10\,000 K$ $\qquad \frac{T}{\left|\frac{dT}{dr}\right|} \sim 10^8 \text{ cm} \sim 1000 \text{ km}.$

- $\boxed{\text{How much is } \ell ?}$ $\qquad$ Pour $\rho = \langle \rho_\odot \rangle = 1.4 \text{ g.cm}^{-3}$

et $\kappa = 1 - 10 \text{ cm}^2 g^{-1}$, on a $\ell = \frac{1}{\kappa\rho} \sim 0.1 - 1$ cm

LTE is verified $\qquad \ell \ll \frac{T}{\frac{dT}{dr}}$ $\qquad$ provided one is deep enough in the atmosphere

(Indeed @ $T = 6000K$ $\ell \gtrsim 1000$ km !)

Stellar interiors : one can show that anisotropy is very weak

- let's develop I in power of $\cos\theta$

$$I(\theta) = \underbrace{I_0}_{\text{isotropy}} + \underbrace{I_1 \cos\theta}_{\text{deviation from isotropy}} + \underbrace{I_2 \cos^2\theta + \ldots}_{\text{negligeable}}$$

- Equation of transfer for a plane-parallel atmosphere (negligible curvature)

$$ds = \frac{dr}{\cos\theta}$$

$$\cos\theta \frac{dI_\nu}{dr} = \rho(j_\nu - \kappa_\nu I_\nu)$$

Replacing I by its development, one shows:

(will be done during exercices)

$$\frac{dI_{n-1}}{dr} = -\kappa\rho I_n \quad \text{for} \quad n > 0$$

$$\Rightarrow \frac{I_{n-1}}{R} \sim \kappa\rho I_n \Rightarrow \frac{I_n}{I_{n-1}} \sim \frac{1}{R\kappa\rho} \sim 10^{-10}$$

The series converges rapidely

$$I = I_0 + I_1 \cos\theta$$